What you should learn from Recitation 3: First order linear ODE

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Disclaimer

- These slides are designed exclusively for students attending section 1, 2 and 3 for the course 640:244 in Fall 2013. The author is not responsible for consequences of other usages.
- These slides may suffer from errors. Please use them with your own discretion since debugging is beyond the author's ability.

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$$y'(t) + p(t)y(t) = g(t)$$

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So integrating both sides, you will get $I(t)y(t) = \int I(t)g(t)dt$, which implies our formula.

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Exercise Check the statements above.

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- If $y_1(t)$ is a (particular) solution

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Exercise Prove this theorem.

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Exercise Test this trick on all the homework problems in Section 2.1.

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$$\begin{cases} ty' + (t-1)y = -e^{-t}, \\ y(\ln 2) = 1/2 \end{cases}$$

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NOTE: PENALTY will be issued if you mess up with the logarithm.

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3. Get the general solution of the ODE.

Summary: Standard Procedures

1. Before you apply any formula, remember to transform the equation into the standard form, i.e. the coefficient of y' shall be 1.
2. Compute the integrating factor based on the data given by the standard form.
3. Get the general solution of the ODE.
4. Use the initial value to decide the arbitrary constant.

Quiz Problem 2

Give the general solution of the ODE

$$xy' = \sqrt{9 - y^2}.$$

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Since $y = 3\sin u$, $u = \arcsin\frac{y}{3}$. So the same result is recovered.

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Either way, you have

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$$y(x) = 3\sin(\ln|x| + C).$$

Graded Homework Problem: 2.2.7.

Solve the differential equation

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}.$$

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You cannot expect to further simplify this. This will be the answer.

Modeling Problem 2.3.6

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- The volume of the flowing out liquid within Δt

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- The volume change brought by the height change Δh

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And as a common sense, $g = 9.8$.

Modeling Problem 2.3.6

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which can be simplified as

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Use your calculator to obtain the T.

Modeling Problem 2.3.8

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$$S(0) = 0$$

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$$\frac{dS}{dt} = k + rS.$$

- So this sentence actually defines an initial value problem:

$$\begin{cases} \dfrac{dS}{dt} = k + rS \\ S(0) = 0 \end{cases}$$

Solving this initial value problem, one gets

$$S(t) = \frac{k}{r}(e^{rt} - 1),$$

which answers Part (a).

Modeling Problem 2.3.8

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Modeling Problem 2.3.8

Both Part (b) and Part (c) needs $S(40) = 1,000,000$, where

- Part (b) fixes $r = 7.5\%$ and asks what k should be;
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Modeling Problem 2.3.8

Both Part (b) and Part (c) needs $S(40) = 1,000,000$, where

- Part (b) fixes $r = 7.5\%$ and asks what k should be;
- Part (c) fixes $k = 2000$ and asks what r should be.

Use your calculator to obtain a result.

The End